Caribbean STEM Olympiads

Math Olympiad Sample Questions

Level I – Sample Questions and Answers

Consumer Arithmetic

Q1: *Julia purchased a new car at $18,000. If the sale price is 25% less than the original price, what was the car's original price?*

A1: $24,000

Q2: *The simple interest earned on a certain sum of money in 5 years at 9% per annum is $4016.25. Calculate the sum of money.*

A2: $8925

Q3 (Multiple choice): *John purchases a table on a hire purchase plan with a down payment of $300 and 12 monthly payments of $24 each month. The regular cash price of the same table is $500. How much would John have saved if he purchased the table using cash?*

A3: $212
$56
$88
$288

Number concepts

Q1: *Water exists in the liquid state between* 0 °C *and* 100 °C. *Given that the formula to convert degrees Celsius (C) to degrees Fahrenheit (F) is* $F = \left(C \times \frac{9}{5}\right) + 32$, *how many integer temperature values between 0 °C and 100 °C (excluding* 0 °C *and* 100 °C*) can be converted to whole numbers in degrees Fahrenheit? Show/explain your reasoning for partial credit.*

A1: 19

Q2 (Multiple choice): *A positive number n is 30% of five thirds of its cube. What is the value of n?*

A2: $\sqrt{2}$

2

$\sqrt[3]{2}$

$\frac{1}{\sqrt{2}}$

Q3: *Simplify the expression* $\left(\frac{a^{\frac{1}{2}}\sqrt{a^3b^4\sqrt{c}}}{c^{-\frac{1}{4}}}\right)^{-1}$*, expressing your answer in the form* $\frac{1}{a^Ab^Bc^C}$ *where A, B and C are positive numbers.*

A3: $\frac{1}{a^2b^2c^{\frac{1}{2}}}$

Fractions and decimals

Q1: *Reduce the following fraction to its lowest terms:* $\frac{3}{4+\frac{2}{1-\frac{1}{4}}}$.

A1: $\frac{9}{20}$

Q2 (Multiple choice): *Compute the following expression, giving your answer as a proper fraction.*

$$\frac{1}{4} + 0.125 - 16^{-1}$$

A2: $-15\frac{5}{8}$

$\frac{5}{16}$

$\frac{7}{16}$

$\frac{1}{12}$

Q3: *Evaluate* $\frac{\frac{2}{5}+\frac{1}{6}}{\frac{12}{7}\times\frac{9}{4}}$.

A3: $\frac{119}{810}$

Statistics & Probability

Q1: *Leroy plays in a series of basketball matches. He scored 19 points on Monday, 23 points on Tuesday, 13 points on Wednesday, and 5 points on Thursday. On Friday, Leroy scored 3 points more than the mean (average) number of points he scored on the first four days. How many points did Leroy score in all?*

A1: 78 points

Q2: *A fair coin is flipped twice. What is the probability of getting at least one head?*

A2: $\frac{3}{4}$

Algebra

Q1: *Evaluate the expression* $(\frac{3a+2b}{2})$ *when* $a =- 3$ *and* $b =- 4$.

A1: -8.5

Q2: *Solve the following simultaneous equations for x and y.*

$2x + y = 12$
$6x + 5y = 40$

A2: $x = 5,\ y = 2$

Q3 (Multiple choice): *How many real solutions does* $3x^2 - 2x + \frac{1}{3} = 0$ *have?*

A3: 0

cannot be determined

1

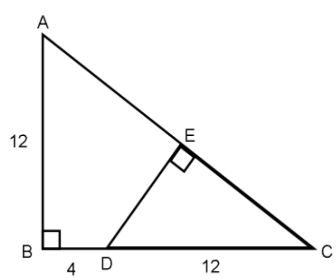
2

Q4: *A car travels from Town A to Town B moving at a constant speed. The car starts at Town A and covers a distance of 1500 metres in 1 minute 40 seconds. The car stops for 15 seconds and then resumes its journey. If the total journey took 3 minutes 30 seconds, what is the distance between the two towns?*

A4: 2,925 m

Geometry

Q1: *In the diagram below,*



(a) *What is the length of the line AC?*

(b) *What is the product of the lengths DE and AC?*

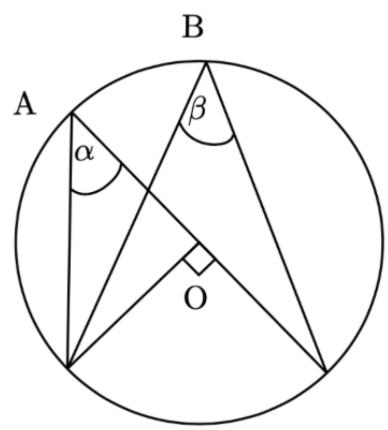
A1(a): 20 units

A1(b): 144 units^2

Q2: *The length of the diagonal of a square is 1 meter. Compute the area of this square.*

A2: $\frac{1}{2}m^2$

Q3: *You are given the circle below. The angle β is bisected by a line OB (not shown). If the diameter of the circle is D, what is the length of the arc AB?*



A3: $\frac{\pi D}{8}$units

Level II – Sample Questions and Answers

Algebra

Q1: *We are given that* $x^4 - y^4 = 13,\ x^2 + y^2 = 4.$ *What is the value of* $(x + y)^2$?

A1: $(x + y)^2 = 4 \pm \frac{1}{4}\sqrt{87}$

Q2: *For what positive values of k will the equation* $kx^2 - (k + 1)x - 1 = 0$ *have exactly one real solution?*

A2: none

Q3 (Multiple choice): *Which of the following values of x satisfies the inequality* $x^2 + 8x + 15 < 0$?

A3: $x = -4$
$x = -2$
$x = -5$
$x = -3$

Q4: *Find the two values of x for which* $x^{1+\log_{10} x} = 10^4 \cdot x.$

A4: $x = 10^2$ and $x = 10^{-2}$

Geometry

Q1: *Find the equation of the sphere whose diameter has endpoints (-2, 3, 1) and (0, 5, - 1).*

A1: $(x + 1)^2 + (y - 4)^2 + z^2 = 3$

Q2 (Multiple choice): *A solid wooden cube is painted red on the outside. The cube is then cut into 27 smaller identical cubes. What fraction of all of the smaller cubes' sides are painted red?*

A2: $\frac{1}{6}$

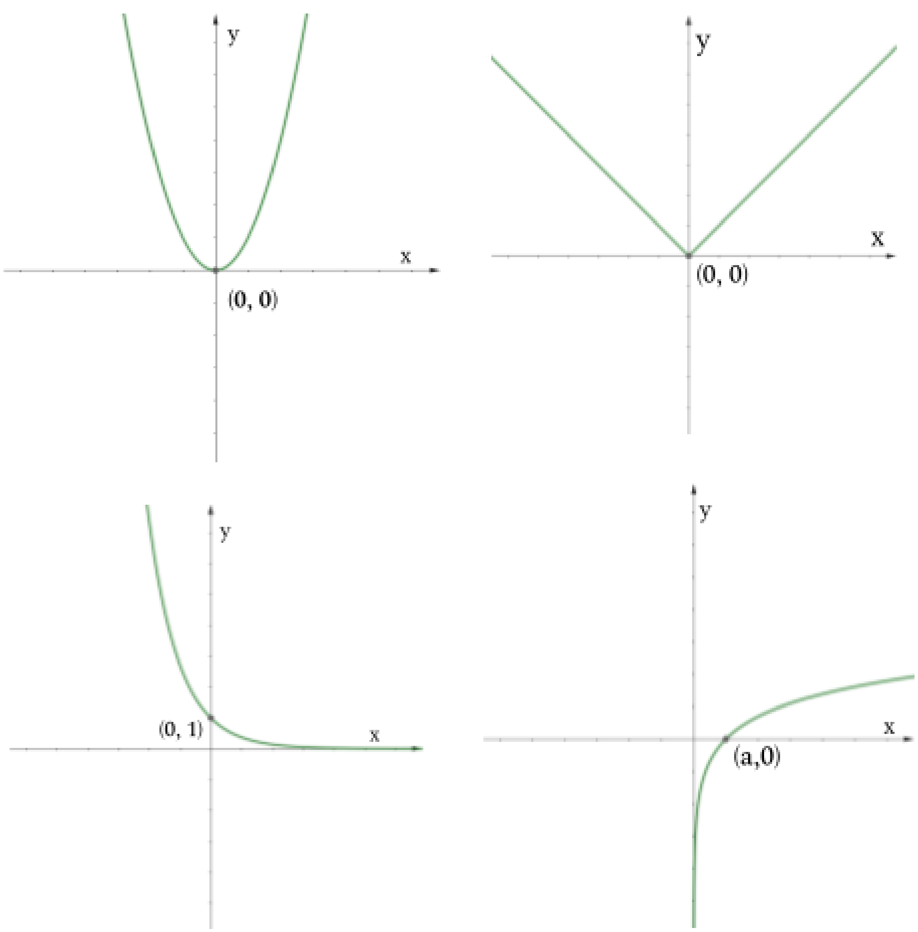
$\frac{2}{7}$

$\frac{1}{3}$

$\frac{1}{2}$

Q3: *Rapid Plot! Sketch the following in 1 minute (note that a is a positive integer):* $y = ax^2$, $y = |x|$, $y = e^{-ax}$, $y = log(\frac{x}{a})$.

A3:



Trigonometry

Q1: *If cos (A) =* $\frac{3}{5}$ *, find sin (2A).*

A1: $\frac{24}{25}$

Q2: *If* $\tan(x+y)=\sqrt{3}$ *and* $\tan(x-y)=\frac{1}{\sqrt{3}}$ *, where* $(x+y) \in (0°, 90°)$ *and* $x > y$*, find x and y.*

A2: $x = 45°,\ y = 15°$

Q3 (Multiple choice): *Given that* $\sec\theta = \frac{7}{3}$*, find the value of* $\sin 2\theta$*.*

A3: $\frac{12\sqrt{10}}{49}$

$\frac{4\sqrt{10}}{3}$

$\frac{3}{7}$

$\frac{2\sqrt{10}}{7}$

Vectors

Q1 (Multiple choice): *Consider a row vector P, consisting of 6 elements, and a column vector Q, also consisting of 6 elements. In which order can vector multiplication be performed?*

A1: PQ only

QP only

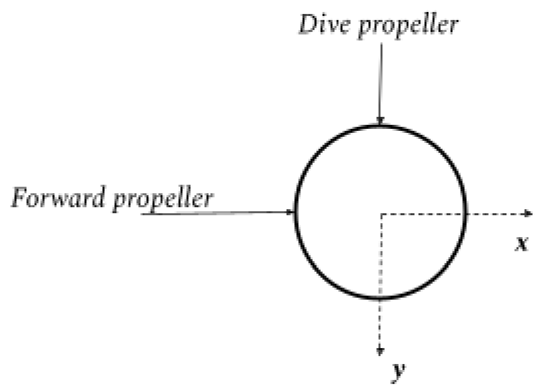
Both ways

Neither

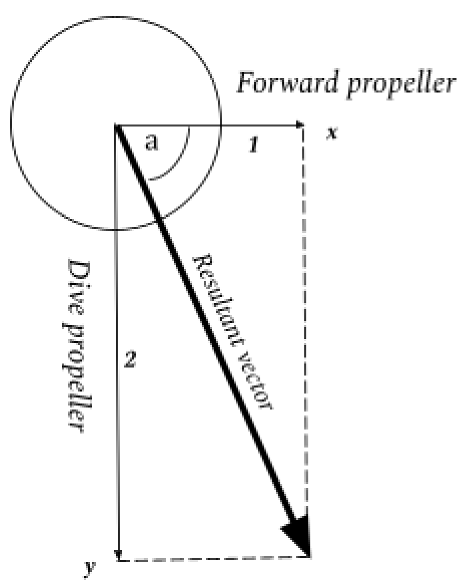
Q2: *If a = 2i + k and b = -3j + 2k (where i is the unit vector along the x axis, j is the unit vector along the y axis, and k is the unit vector along the z axis), find the unit vector u that is opposite in direction to v = 2a - b.*

A2: $u = \ -\frac{4}{5i} - \frac{3}{5j} + \ 0k$

Q3: *A spherical underwater robot has 2 propellers: one to dive and one to move forward. The dive propeller applies a net force (beyond buoyancy) that is twice as strong as the forward propeller. Complete the diagram to show the direction of the robot's movement.*



A3:



Resultant vector magnitude: $\sqrt{5}$

Resultant vector angle below horizontal axis: $a \ = \ tan^{-1}(2)$

Exponentials and Logarithms

Q1: *The decibel (dB) is a unit of measurement for the loudness of sounds. A 10-fold increase in loudness corresponds to an increase of 10 dB (100-fold is +20 dB, 1000-fold is +30 dB, and so on).*

(a) *Write the relation between the fold-loudness of a sound and its decibel value. You may use* S_0 *and* S_1 *to represent the sound intensities.*

(b) *You're sitting at a music mixing board and are told to make the speaker output twice as loud. How many decibels should you increase the output level of the speaker by?*

A1(a): $10log\frac{S_1}{S_0}\ dB$.

A1(b): Decibel output should be increased by $10log2$

Q2: *If* $xy^m = yx^3$, *then solve for m.*

A2: $m = 1 + 2log_x y$

Q3 (Multiple choice): *Suppose that the population of a colony of bacteria increases exponentially. If the population at the start is 300 and 4 hours later it is 1800, how long will it take for the population to reach 3000?*

A3: $10\frac{ln\ 4}{ln\ 6}$

$4\frac{ln\ 10}{ln\ 6}$

$4\frac{ln\ 6}{ln\ 10}$

$6\frac{ln\ 10}{ln\ 4}$

Q4: *Find the positive integer solution of*

$log_3 x\ log_4 x\ log_6 x = log_3 x\ log_4 x + log_4 x\ log_6 x + log_3 x\ log_6 x$.

A4: $x = 72$

Calculus

Q1: *A girl sits on a cliff 20 meters above the ground. She throws a tennis ball vertically upwards from the cliff. After t seconds it has height s above the ground which follows:*

$$s = 20 + 50t - t^2$$

What is the maximum height of the ball?

A1: 645m

Q2: *Find the function, y(x), that satisfies the differential equation:*

$$\left(\frac{dy}{dx}\right)^2 = \frac{1-y^2}{1-x^2}$$

A2: y = sin[arcsin x + C]

Q3 (Multiple choice): *Find the second derivative of sin(x^2 – 4) with respect to x.*

A3: $4x^2 sin(x^2 - 4) - 2cos(x^2 - 4)$

$-4x^2 sin(x^2 - 4) + 2cos(x^2 - 4)$

$2x^2 sin(x^2 - 4) + 2cos(x^2 - 4)$

$2x^2 sin(x^2 - 4) + 4cos(x^2 - 4)$

Level III – Sample Questions and Answers

Series and Sequences

Q1: *What is the sum of the infinite series* $\frac{7}{4} - \frac{7}{4^2} + \frac{7}{4^3} - \ldots$?

A1: $\frac{7}{5}$

Q2 (Multiple choice): *The sequence* $x_n = \frac{1}{n}$, $n \in \mathbb{Z}^+$ *is:*

A2: Bounded and Convergent
Bounded and Divergent
Unbounded and Convergent
Unbounded and Divergent

Euclidean Geometry

Q1 (Multiple choice): *Consider a circle with three distinct points labelled A, B, and C on its circumference. What is the minimum number of straight lines/line segments which must be drawn to locate the center of this circle?*

A1: 6 lines
2 lines
3 lines
4 lines

Q2: *Let A be a point in* $\mathbb{R}^3$ *with cartesian coordinates* $(x_0,\ y_0,\ z_0)$. *A can be expressed in spherical coordinates as* $(\rho_0,\ \theta_0,\ \phi_0)$. *The following equations can be used to convert from spherical coordinates to cartesian coordinates:*

$$\begin{cases} x = \rho \sin\phi \cos\theta \\ y = \rho \sin\phi \sin\theta \\ z = \rho \cos\phi \end{cases} \quad \text{for } \rho \geq 0,\ \theta \in [0, 2\pi) \text{ and } \phi \in [0, \pi).$$

Find $\phi_0(x_0, y_0, z_0)$.

A2: $\phi_0 = arctan\left(\frac{\sqrt{x_0^{\ 2} + y_0^{\ 2}}}{z_0}\right)$

Calculus

Q1: *Evaluate* $\int_0^{\frac{\pi}{2}} \frac{cos\ \theta}{1 + sin\ \theta} d\theta$.

A1: $ln\ 2$

Q2 (Multiple choice): *Integrate* $\int x\ cos\ x\ dx$.

A2: $x\ cos\ x\ -\ sin\ x$

$x\ sin\ x\ -\ cos\ x$

$x\ sin\ x\ +\ cos\ x$

$x\ cos\ x\ +\ sin\ x$

Q3: *Simplify* $\int sin^3(t) cos^2(t)\ dt$ **.**

A3: $-\frac{cos^3(t)}{3} + \frac{cos^5(t)}{5} + C$

Vectors & Matrices

Q1: *Find the determinant of the matrix:* $\begin{bmatrix} 1 & -4 & 7 & 5 \\ 0 & 3 & 1 & -6 \\ 1 & -1 & 7 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$

A1: -6

Q2: *The eigenvalues,* λ*, of an* $n \times n$ *matrix, A, can be found by solving the equation* $det(A - \lambda I) = 0$ *where I is the* $n \times n$ *identity matrix. Find the eigenvalues of* $A = \begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix}$.

A2: $\lambda_+ = \frac{5+i\sqrt{3}}{2}$ and $\lambda_- = \frac{5-i\sqrt{3}}{2}$

Complex Numbers

Q1: *The complex number, z, can be represented by a vector in the complex plane. What is the effect on the vector representing z, in the complex plane, of multiplying z by* $re^{i\theta}$, $r \in (0, \infty)$, $\theta \in [-\pi, \pi)$?

A1: Multiplying z by $re^{i\theta}$ will make the vector r times as long and rotate it an angle of θ counterclockwise around the origin.

Q2: *Find the purely real solution to* $4z^2 + (8 + 4i)z + (3 + 2i) = 0$.

A2: $z = -\frac{1}{2}$

Differential Equations

Q1: *Find the general solution for the differential equation* $\frac{dy}{dx} = x + xy^2$.

A1: $y = tan\left(\frac{x^2}{2} + C\right)$

Q2 (Multiple choice): *Solve the differential equation* $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 0$ *where* $x(0) = 0$ *and* $\frac{dx}{dt}(0) = 5$.

A2: $5e^{-t} - 5e^{6t}$

$5e^{-t} + 5e^{6t}$

$5e^{-2t} + 5e^{-3t}$

$5e^{-2t} - 5e^{-3t}$